PART I
Justifying the Combinatorial Hierarchy

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NOTE: The following paper was put together in haste, based on more careful work done in England for ANPA 12. Unfortunately, the reader must be warned that the paper has not been edited and some typographical errors may have crept in.

I. Introduction

Over the last few years it has become increasingly apparent that the combinatorial hierarchy was simultaneously central to the work ANPA members have done in discrete physics and not well understood. Most of the striking (and unfortunately subtle) computations that have been done rest upon the importance of the combinatorial hierarchy and its properties. Yet those from outside ANPA, and even some members of the group, have questioned the justification of the use of this abstract mathematical structure. Its interpretation has been the subject of considerable speculation and discussion. Worse, the form in which these "mathematical foundations" has been derived and presented has been varied. My own efforts seem nothing short of incomprehensible to others! This problem clearly has need of a frontal attack.

In this paper, I make a start at a justification of the combinatorial hierarchy. Much of what I have to say is motivated by a particular "philosophical" point-of-view. However, I do not believe that the motivating assumptions are too difficult to accept. If the reader finds them abhorent, I suggest that consideration be given to alternative philosophical points-of-view that might equally well motivate this discussion. In other words, I suspect the assumptions are sufficiently general as to be come upon in a number of ways and in particular by thinking about the problem mentioned above.

Along similar lines, the argument which I shall give depends on a particular derivation of the combinatorial hierarchy for its force of exposition. It is a conjecture of mine that any particular form for deriving the combinatorial hierarchy, or for formalizing its generation, can be shown equivalent to the one used here. For this reason, I will refer to this formulation -- which appeared in On the Fine Structure of Hydrogen, as the "canonical derivation." It is repeated in Section VI.

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II. Philosophical Remarks

The combinatorial hierarchy work attempts to describe natural phenomena with as little *a priori* structure as possible. I have long taken the point of view that it is the combination of two, possibly incompatible, structures or languages that leads to the appearance of most interesting phenomena. In the case of physics, it is the structure of "physical reality" -- the object language -- and the structure of the description -- the formal language -- that accounts for much of what we "observe".

This is not a new idea. It is well understood in psychology, anthropology, linguistics, and logic. If an individual has no concept of color, they may experience the world in black and white or shades of grey. An individual whose only explanations of events is in terms of demons will observe events compatible with demons as the causative force. The key point is that observed structure may come from either the descriptive language or from its limitations.

Unfortunately, the interaction between multiple structures of this type has rarely been given consideration in physics except in foundations arguments in which "theory laden" language has sometimes been discussed. I know of very few cases in which any attempt has been made to formalize the situation. While we will not attempt to formalize the effort here, we do claim that such an attempt has been a characteristic of the combinatorial hierarchy work all along.

In the following sections, we first describe a structural "skeleton" for "reality" -- that untouchable, out there. No questions of existence are entertained. This is intended less as a statement of ontological position, than as a gedanken experiment. We then engage in a similar exercise regarding the structure of the practice of physics, as approached through its language. Finally, we examine the interaction between the two.

III. Deducing A Possible Structural Skeleton for "Reality"

Lacking contrary evidence, we treat "physical reality" as having minimal *a priori* structure, try instead to justify the few assumptions we do make, and to derive as much as possible from those assumptions. We assume from the outset that "reality" is simple (easy to understand if we only knew what it was) and universal -- that God does not play tricks and didn't engage in multiple, incompatible efforts.

- **A process point of view**
  - required in order to avoid problems with trying to create evolving systems from static ones
  - structure is generated from process
  - the characteristic numbers of this structure are significant
- **A discrete point-of-view**
  - demanded by observed properties of the universe
- **The process is self-reproducing**
  - otherwise a more fundamental "cause" is required, leading to an infinite regress
- **If universal, then must be self representing too!**

If the activity of modeling physical reality is itself a part of physical reality, it must be possible for structure to represent itself, possibly with some loss of resolution. But this already means that the structure is hierarchical.

- **The structures generated, and the process, must convey information, and so have an information theoretic represention**
  - they must have observable consequences

We take the structure to be of an information theoretic character: in other words, the fundamental units of this structure are bits of information. If information is to be conveyed by this structure, then this assumption adds nothing. In this regard, a number of physicists have come to this point-of-view, but have not had great success in producing a coherent and explanatory theory that is purely information theoretic.

- **The elements of the process behave as both operators and operands.**
  - there is only one set of fundamental elements
  - The objects formed from fundamental elements are the subsets closed under the operation of discrimination.

Again, I would argue that this is as simple as could be desired.

- **It must be hierarchical in structure**
  - demanded by "scale invariant" conservation laws
  - self reproducing structures are often, perhaps always, hierarchical

By this we mean that it has a level structure in which each level is similar to the others and each element of the structure is related to elements at other levels in such a way that one can not determine the level other than by the number of elements at any level. This idea is familiar as both successive embeddings and as a hierarhical partitioning of a space.

Because of the hierarchical character of these structures, any property is level independent. Since the dicrete structure has a definite minimum unit, we put "scale invariant" in quotes.

- **Rapidly growing complexity per level**
  - demanded by the complexity of the world
  - rich and non-polynomial

The process character suggests that a particular property of the levels of the hierarchy is the fundamental forces. The diversity in relative strengths of these forces suggests a non-polynomial complexity. Certainly it would be possible to fit a polynomial curve to the relative strengths. However, the fact that physical processes also demonstrate a random character suggests that the problem of predictability is NP-complete. Nature "computes" its next step in a manner that is more complicated than we can analyze.

- **The complexity is combinatorial**
  - it is not exponential

Because the system is discrete, the process which generates the hierarchy is characterized by a combinatorial rather than an exponential formula. Actually, this is just another way of saying "if a structure can be created by the combination of existing structures, it is." This keeps everything simple by not invoking rules about what is forbidden.

- **Non-trivial structure implies discrimination over Z_2.**
  - a principle of distinguishability or equivalence is required
  - for a binary system we call that discrimination

The operation of discrimination is just a simple notion of equivalence: discrimination leads to the set of null elements if and only if the compared sets are identical. Most often, this comparison is performed on ordered sets which can be represented as ordered binary strings.

- **The properties of the structure must be "approximately continuous"**
  - Mathematical correspondence principle: this demands something like $2^n - 1$.

The world has long been treated as though it were a continuum. If this is not true (and it appears that quantum mechanics denies it), then we must have a mathematical structure which is "approximately continuous" in its properties. Herb Doughty pointed out sometime ago that a structure like $2^n - 1$ with successive embeddings can approximate the continuum. In particular, it is not generally possible to decide that no intermediate points exist (my apologies to Herb if I don't have this quite right). As I recall, the claim is actually stronger: any mathematical space capable of approximating the continuum can be represented with such an embedded structure.

Such a structure has no inherent notion of temporal evolution or causal structure. The generative process exists as a more primitive notion than space and time.

- **The number of elements *n* at level *i+1* is given by recursion:**

$$n_{i+1} = 2^{n_i} - 1$$

IV. The Structure Implied by the Language of Physics

As the formal language, therefore, we require a structure capable of having a causal structure. However, it must also have enough structure compatible with the structure of "reality" so that information can be given a representation in either (up to a point).

- **Non-singular operators are required**
  - The "physical" character of representation demands non-singular operations (i.e. invertible maps between elements - these are the internal coordinant automorphisms)

- **A causal structure must be supported**
  - The vector space has a complete and orthogonal basis
  - It has a representation as square matrices
  - With Z_2, n^2 operators per level is then the only answer.

One way to support a causal structure -- it may be both sufficient and necessary -- is to have a vector space with a complete and orthogonal basis. It is then possible to obtain evolution operators, and, from this, a notion of temporality.

- **This structure starts with a two-dimensional complete, orthogonal space**
  - this is the simplest space with structure

The traditional structure of the combinatorial hierarchy requires a series of spaces which are represented by 2x2, 4x4, 16x16 matrices, etc. One might ask why not start with 3x3 or 5x5. Again, we take this structure to be as simple as possible. This means that the simplest such structure is 2x2.

- **Separability of hierarchical levels is required**
  - there must be no confusion of object types
  - Seen top-down, this is a partitioning into maximally disjoint subspaces of equal dimension
  - This means that the next level will be 4x4, then 16x16, etc.
  - This means that there is a notion of locality.

- **The number of elements *m* at level *i* is given by recursion:**

$$m_i = m_{i-1}^2$$

V. Interaction Between Structures

- **Unique self-representation is limited to four levels - thereafter we have no novelty.**

The two structures can only be mapped in such a way as to preserve unique objects in the "physical" structure up to four levels. Thereafter the unique objects in the physical structure become confused with each other.

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- **The characteristic numbers of the structures so generated are appropriate for physical interpretation.**

A hierarchy has certain characteristic numbers which are formed from the cardinality of each level and from the number of objects which remain invariant and unique under the operation which defines the hierarchy. Having generated such structures, it is then up to an analysis of the research results to determine if the numbers are meaningful. The now long list of "intriguing numerology" that has been published represents an unlikely coincidence.

VI. The Canonical Derivation

The Combinatorial Hierarchy is generated from two recursively generated sequences. The first is governed by the recursion formula

$$\mathbf{n}_{(i+1)} = 2^{n_i} - 1,$$

(a formula familiar to those who have studied the Mersenne primes), and begins with the term **n = 2** leading to the sequence 3, 7, 127, $2^{127} - 1$,... The cumulative cardinals of this series (ignoring the initial term) also form a series which has interpretive significance, namely 2, 3, 10, 137, ~1.7016... * 10e38 + 137,...

The second recursively generated sequence is governed by the formula

$$m_{(i+1)} = m_i^{\ 2}$$

also beginning with the term **m = 2** and leading to the sequence 2, 4, 16, 256, 65536,...

These two sequences have various justifications. Perhaps the clearest presentation has been given by Clive Kilmister (correspondence to H.P.Noyes, date Oct.16, 1978), paraphrased here as follows:

Definition: By a **combinatorial hierarchy** is meant a collection of levels related as follows:

a) the elements at level L are a basis of a vector space V/Z_2

b) the elements at level L+1 are non-singular (i.e. invertible) linear operators mapping V/Z_2 into V/Z_2

c) each element A at level L+1 are mapped to a subset S of the elements at level L by the correspondence: the proper eigenvalues of A [i.e., Av = v] are exactly the linear subspace generated by S.

d) each element at level L+1 is chosen independent, allowing the process to be repeated for level L+2, L+3, L+4, ...

Theorem 1: There *exists* a *unique* hierarchy (up to isomorphism) with more than 3-levels and it has the following successive numbers of elements:

$$2,\ 3,\ 7,\ 127,\ 2^{127} - 1$$

and terminates at Level 4 due to the fact that the operators have m^2 elements if the vectors are m-fold, and 2^n (required for V/Z_2 increases too fast.

One can think of the number of elements at each level of a combinatorial hierarchy as being the number of subsets of a set of n things. Given the definition of a hierarchy over the field Z_2, this generates the first sequence mentioned above. Over a finite field V, all operators which map V into V may be thought of as permutations in that they map elements of V into other elements of V and this map can be given a pairwise representation. Note then that is it possible to think of the number of independent operators as simply the number of permutations of n things taken two at a time (i.e. exchange of a and b corresponds to mapping a to b) or m^2 total at any level of the hierarchy, given m at the previous level. Clearly, this forms a complete set of independent operators - all other permutations of n things can be created by successive application of this set of permutation operators. This generates the second sequence mentioned above which governs a combinatorial hierarchy.

Mapping the first sequence onto the second and treating the second sequence as independent basis strings, one finds that the mapping is not uniquely possible beyond the fourth term or level. Thus the first sequence can not be a coordinate basis beyond level four of the hierarchy. The inter-relationship forms a **stop rule** which signals the end of global novel structure generation, although not necessarily the end of generation and redundant structure. In other words, if one codes the algorithm as a program, there will be a point at which no further novelty will result, although the program need not halt altogether. The cardinals of elements and operators at each level will be determined by the two sequences given above. As noted in Theorem 1, any two runs of the program will produce hierarchies which are isomorphic, even though the particular evolution may differ and the particular objects used to represent the elements may differ. For this reason, and since the details of evolution are generally not significant for us, we refer to **The Combinatorial Hierarchy.**

Part II
THE ELECTRON-PROTON MASS RATIO

I. Introduction

The computation of the electron-proton mass ratio produced by Frederick Parker-Rhodes was based on the combinatorial hierarchy numbers, in particular 137. However, the model he used was strictly a continuum probability model. He evaluated the distribution of a charge in order to come up with certain weighting factors. In this part of the paper we show how the same calculation can be given a purely combinatorial basis and briefly discuss the model of the proton and electron that this basis implies.

NOTE: Due to typographical problems, I will use *p* for the constant "pi" and *a* for alpha.

II. The Electron Mass

Let $a_1 = 137*2p$. We interpret 137 as an "angular" frequency (in the probability sense), suggesting that the usual fine structure already has a $2p$ embedded in it. The value a_1 is the "linearized" counterpart to the first order approximation of the fine structure constant.

Model electron "mass" as:

1) primarily due to Coulomb events (a_1).

2) due to a level 2 self-interaction: 6 of every 7 events are indistinguishable between the electron and itself)

3) the effects due to virtual electron "generation" are with respect to that without self-interaction and so we normalize with respect to that effect.

The electron picture suggested by this model is that of a "bag" of 137 different randomly occuring events, but for which 6 out of 7 of these event are equivalent under some set of operations (namely those defined at Level 2 by the combinatorial hierarchy). The boundary of this "bag" defines the electron Compton wavelength in terms of the possible space-time parameters than can be consistently given to these events while maintaining a causal structure.

Although this is a Level 2 process, we write it out in Level 1 "units" -- i.e. since there are three Level 1 events for every seven Level 2 events, multiply by 3/7 -- this will allow us to combine the computations for electron and proton:

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$$m_e = (3/7)*(a_1)*[1-(6/3*7)^N] / [1-(6/3*7)] \qquad (i)$$

where N is the degree of self interaction (this is an important interpretive point -- see Appendix A below).

III. The Proton Mass

Now treat m_p, the proton observed "mass", as:

1) Consisting of two parts:

   - a "fundamental" "mass" m_p(fundamental)

   - a portion due to Level 1/Level2 coupling:

     m_p(coupling) = 3/7 m_p

2) There is no self-interaction term (the proton is always distinguishable from itself)

3) As with the electron, normalize with respect to the level 2 term as given for the electron.

Unlike the model of the electron, this proton model treats the proton as more fundamental. It is entirely due to coupling between Level 1 and Level 2.

Write this as

$$m_p(\text{fundamental}) = [m_p - m_p\ (\text{coupling})] / m_p\ (\text{virtual})$$

or

$$m_p(\text{fundamental}) = m_p * [1-(3/7)] / [1-(6/3*7)] \qquad (ii)$$

Combining (i) and (ii) we obtain:

$$m_p/m_e = ([1-(6/3*7)] / [1-(3/7)]) / \{(3/7)(1/2p)(1/137)[1-(6/3*7)^N]/[1-(6/3*7)]\} \qquad (iii)$$

which, for N=3, is algebraically identical to:

$$m_p/m_e = 137p / (3/14)[1+(2/7)+(4/49)]*(4/5) \qquad (iv)$$

$$= 1836.151497$$

as derived by Parker-Rhodes using integrals. Note that the value of N just corresponds to the number of expansion terms in the series solution to one of the Parker-Rhodes integrals.

Appendix A: A Note Regarding Self-Interaction

The combinatorial formula $1-r/(1-r^n)$ has an expansion with which we are of course familiar, provided r is less than unity. Reinterpret this as follows. Suppose that a certain process P has occurrence probability r. Then the probability that P will recur given sampling with replacement is r^n. This corresponds to the branching probability computed in a Feynman diagram for self-interaction terms. The complement 1-r is the probability of NOT P and that of $1-r^n$ is that of NOT (P self interacting n times). The ratio

Appendix B: A Comment on A "New" Correspondence Principle

While I don't believe that the kind of "correspondence principle" that is espoused between quantum mechanics and classical mechanics should apply to the combinatorial hierarchy work, I believe that a truly fundamental theory should be able to explain the successes of earlier theories. Typically, this means understanding earlier efforts in terms of approximations and even interpretational or experimental errors. These will often arise because the theorist attempts to force-fit experimental data into a given framework.

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